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# Calculation of the Secrecy Capacity

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## INSTALLATION

You can install the package via pip

```
1 pip install secrecy-capacity
```

If you want to install the latest (unstable) version, you can install the package from source

```
1 git clone https://github.com/klb2/secrecy-capacity-calculation.git
2 cd secrecy-capacity-calculation
3 git checkout dev # only if you want to install the unstable version
4 pip install .
```

You can test, if the installation was successful, by importing the package

```
1 import secrecy_capacity
2 print(secrecy_capacity.__version__)
```



## EXAMPLES

In the following, some simple usage examples are given.

You can also find example scripts in the `examples/` directory in the repository.

## 2.1 Loyka Algorithm

The first algorithm that you can use to calculate the optimal transmit covariance matrix to maximize the secrecy rate is the one from reference [1].

A very simple example is provided below. You only need to create the channel matrices to Bob and Eve and define your power constraint.

```
1 import numpy as np
2 from secrecy_capacity import cov_secrecy_capacity_loyka, secrecy_rate
3
4 # Random generation of 2x2 channels
5 channel_bob = np.random.randn(2, 2) + 1j*np.random.randn(2, 2)
6 channel_eve = np.random.randn(2, 2) + 1j*np.random.randn(2, 2)
7
8 power = 10 # power constraint (linear)
9
10 # Calculate the optimal transmit covariance matrix.
11 # This will take a while for the Loyka algorithm
12 opt_cov = cov_secrecy_capacity_loyka(channel_bob, channel_eve, power=power)
13
14 # If you want to calculate the secrecy capacity for the found covariance
15 # matrix, you can use the secrecy_rate function
16 sec_cap = secrecy_rate(channel_bob, channel_eve, cov=opt_cov)
17
18 print("Optimal covariance matrix:")
19 print(opt_cov)
20 print("Secrecy capacity: {:.3f}".format(sec_cap))
```

## 2.2 Low Complexity Algorithm

The second algorithm that you can use to calculate the optimal transmit covariance matrix to maximize the secrecy rate is the low-complexity one from reference [2].

The usage is very similar to the Loyka algorithm.

```
1 import numpy as np
2 from secrecy_capacity import cov_secrecy_capacity_low_complexity, secrecy_rate
3
4 # Random generation of 2x2 channels
5 channel_bob = np.random.randn(2, 2) + 1j*np.random.randn(2, 2)
6 channel_eve = np.random.randn(2, 2) + 1j*np.random.randn(2, 2)
7
8 power = 10 # power constraint (linear)
9
10 # Calculate the optimal transmit covariance matrix.
11 # This will be way faster than Loyka's algorithm
12 opt_cov = cov_secrecy_capacity_low_complexity(channel_bob, channel_eve, power=power)
13
14 # If you want to calculate the secrecy capacity for the found covariance
15 # matrix, you can use the secrecy_rate function
16 sec_cap = secrecy_rate(channel_bob, channel_eve, cov=opt_cov)
17
18 print("Optimal covariance matrix:")
19 print(opt_cov)
20 print("Secrecy capacity: {:.3f}".format(sec_cap))
```

## 2.3 References



## SECRECY\_CAPACITY PACKAGE

### 3.1 Functions

<code>duplication_matrix_fast(n)</code>	Duplication matrix.
<code>cov_secrecy_capacity_low_complexity(mat_bob, ...)</code>	Optimal covariance matrix (low complexity implementation)
<code>cov_secrecy_capacity_loyka(mat_bob, mat_eve)</code>	Optimal covariance matrix (Loyka's algorithm)
<code>secrecy_rate(mat_bob, mat_eve[, cov])</code>	Calculation of the secrecy rate for given realizations of AWGN channels.

#### 3.1.1 duplication\_matrix\_fast

`secrecy_capacity.duplication_matrix_fast(n)`  
Duplication matrix. Implementation from: <https://www.mathworks.com/matlabcentral/answers/473737-efficient-algorithm-for-a-duplication-matrix>

#### 3.1.2 cov\_secrecy\_capacity\_low\_complexity

`secrecy_capacity.cov_secrecy_capacity_low_complexity(mat_bob, mat_eve, power: float = 1, tol_eps: float = 1e-12)`

Optimal covariance matrix (low complexity implementation)

Calculate the optimal covariance matrix for a fading wiretap channel using the low complexity algorithm from [1].

**Parameters**

- **mat\_bob** (*numpy.array*) – Matrix with the channel realizations of Bob's channels.
- **mat\_eve** (*numpy.array*) – Matrix with the channel realizations of Eve's channels.
- **power** (*float*) – Power constraint at the transmitter.
- **tol\_eps** (*float*) – Tolerance level for the inner algorithm, cf. *Algorithm 1* in [1].

**Returns** `cov` – Optimal covariance matrix which maximizes the secrecy rate.

**Return type** `numpy.array`

## References

### 3.1.3 cov\_secrecy\_capacity\_loyka

`secrecy_capacity.cov_secrecy_capacity_loyka(mat_bob, mat_eve, power: float = 10, t: float = 1000.0, alpha: float = 0.3, beta: float = 0.5, mu: float = 2, eps: float = 1e-10, dirname: Optional[str] = None, return_interm_results: bool = False)`

Optimal covariance matrix (Loyka's algorithm)

Calculate the optimal covariance matrix for a fading wiretap channel using the algorithm from [1].

#### Parameters

- **mat\_bob** (*numpy.array*) – Matrix with the channel realizations of Bob's channels.
- **mat\_eve** (*numpy.array*) – Matrix with the channel realizations of Eve's channels.
- **power** (*float*) – Power constraint at the transmitter.
- **alpha** (*float*) – Parameter  $\alpha$  with  $0 < \alpha < 0.5$  is a percent of the linear decrease in the residual one is prepared to accept at each step, cf. *Algorithm 1* in [1].
- **beta** (*float*) – Parameter  $\beta$  with  $0 < \beta < 1$  is a parameter controlling the reduction in step size at each iteration of the algorithm, cf. *Algorithm 1* in [1].
- **mu** (*float*) – Parameter  $\mu$  with  $\mu > 1$  defines the multiplier in the barrier method, cf. *Algorithm 3* in [1].
- **eps** (*float*) – Tolerance level for the outer barrier algorithm, cf. *Algorithm 3* in [1].
- **dirname** (*str*) – Path of the directory in which checkpoints and log should be saved. If None, no intermediate results will be saved.
- **return\_interm\_results** (*bool*) – If True, the history/intermediate results of the algorithm will be return together with the optimal covariance matrix. **This changes the return structure of the function!**

#### Returns

- **cov** (*numpy.array*) – Optimal covariance matrix which maximizes the secrecy rate.
- **(interm\_res\_norm, interm\_sec\_rate)** (*tuple of list of float*) – **Only returned, when return\_interm\_results == True!** Tuple that represents the history of the algorithm. The norm of the residual is stored in `interm_res_norm`, while the intermediate secrecy rates are in `interm_sec_rate`.

## References

### 3.1.4 secrecy\_rate

`secrecy_capacity.secrecy_rate(mat_bob, mat_eve, cov=None) → float`

Calculation of the secrecy rate for given realizations of AWGN channels.

Assuming AWGN fading channels to both Bob and Eve, this function calculates the secrecy rate for a given fading realization and covariance matrix at the transmitter.

Let  $H$ ,  $G$  be the channel matrices to Bob and Eve, respectively, and  $Q$  the covariance matrix at Alice. The secrecy rate  $R_S$  is then given as

$$R_S = \max \{ \log_2 (I + HQH^H) - \log_2 (I + GQG^H), 0 \}$$

**Parameters**

- **mat\_bob** (*float*, *complex* or *numpy.array*) – Matrix or scalar of Bob’s channel realizations.
- **mat\_eve** (*float*, *complex* or *numpy.array*) – Matrix or scalar of Eve’s channel realizations.
- **cov** (*float* or *np.array*) – Covariance matrix (or transmit power) at Alice. If None, the identity matrix will be used.

**Returns** **sec\_rate** – Secrecy rate  $R_S$  as described above.

**Return type** *float*



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